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United States
Department of
Agriculture

Forest Service

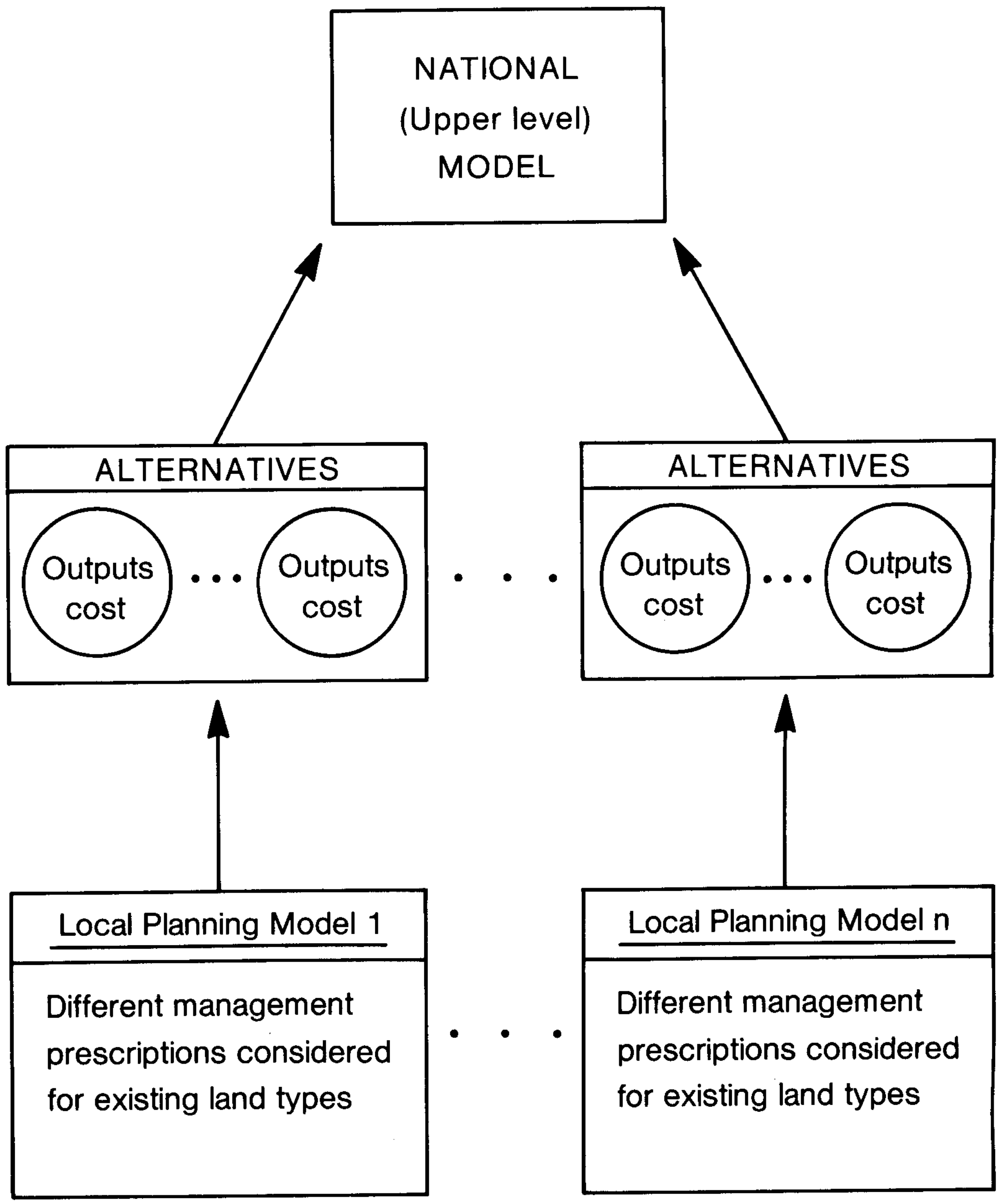
**Rocky Mountain
Forest and Range
Experiment Station**

Fort Collins,
Colorado 80526

**General Technical
Report RM-130**

A Multilevel Optimization System for Large-Scale Renewable Resource Planning

John G. Hof and James B. Pickens



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May 1986

A Multilevel Optimization System for Large-Scale Renewable Resource Planning

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Abstract

This report analyzes and evaluates one approach to utilizing local planning analyses of public renewable resource management agencies as an input in developing large-scale (national) resource management plans. A type of multilevel approach (referred to as the Bartlett-Wong approach) is discussed and is evaluated in a test case. This prototype performed well at the highest level of analysis but was less reliable in terms of its implications for the lower level planning units.

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A Multilevel Optimization System for Large-Scale Renewable Resource Planning

John G. Hof and James B. Pickens

Introduction

Historically, national resource optimization analyses have dealt with very highly aggregated variables and with an extremely low level of resolution. As a result, such analyses could not be validated or monitored. Attempts to disaggregate national planning (optimization) results to local levels also have been generally unsuccessful.

One extreme alternative is to base national planning on simple aggregations of local level optima.[2] However, this alternative is in conflict with the reasons for national planning in the first place: input and/or output constraints (including objectives, etc.) that apply across local planning unit boundaries.

Instead a second alternative—a multilevel approach—is examined here. It utilizes local analyses to generate information for a national analysis that, in turn, can accommodate national concerns and constraints. An example of a use for such an approach is in the development of the USDA Forest Service National (RPA) Assessment and Program.

This report is based on the assumption that the principal purpose of national planning in renewable resource agencies is to select the output mix to be produced and the means of producing it. Although there are many informational outputs of national planning analysis (e.g., price projections, consumption and production projections, resource inventories, and distributional effects on income and employment), this report is limited to efficiency-oriented analysis. It would be impossible to evaluate in one publication all of the different analytical approaches that can be included in national renewable resource planning efforts.

The Problem

For this discussion, define the following variables:

X_1 = budgetary input (cost, ignoring land input)
X_2 = land input
Y_1 = timber output
Y_2 = forage output
f = an implicit production function
Z_i = production factors purchased with budget (X_1)
r_i = factor prices of the Z_i
P_1 = price of timber
P_2 = price of forage.

[2]This means a single aggregation, adding up all of the "preferred" or "final" plans from the local levels. This should be distinguished from an approach where national planning considers a variety of aggregations of different lower level alternatives. Many of the latter sort of aggregations might be possible. For example, if there were 10 different alternative plans for each of 100 local planning units, then there would be 10^{100} different alternative national aggregations.

The "0" superscript is used to indicate a fixed level of the given variable. For this discussion, only two outputs (timber and forage) are considered. It is assumed primary objective of a planning optimization analysis is to locate efficient points—points that are on a production frontier. The optimization problem relevant to public renewable resource planning is somewhat different than the classic economic analysis of production (Silberberg 1978), because one input, land (X_2^o), is not priced and is fixed. The problem is to purchase other inputs (Z_i) with the budget (X_1) so as to produce outputs (Y_j) efficiently. Assuming that there are only two Z's, one way to formulate this problem is

$$\text{Minimize: } X_1 = r_1Z_1 + r_2Z_2 \quad [1]$$
$$\text{Subject to: } Y_1 \geq Y_1^o$$
$$Y_2 \geq Y_2^o$$
$$X_2 \leq X_2^o$$
$$f(Z_1, Z_2, X_2, Y_1, Y_2) = 0.$$

Solving this problem is essentially attempting to locate an efficient point on a particular production frontier (product transformation curve).

Another way to accomplish basically the same thing is to solve the following problem.

$$\text{Maximize: } P_1Y_1 + P_2Y_2 \quad [2]$$
$$\text{Subject to: } r_1Z_1 + r_2Z_2 \leq X_1^o$$
$$X_2 \leq X_2^o$$
$$f(Z_1, Z_2, X_2, Y_1, Y_2) = 0.$$

The P_1 and P_2 simply provide for a given weighted summation of the outputs. Solving this second (revenue maximizing) formulation also amounts to locating points on a production frontier (product transformation curve). If the Y_1^o and Y_2^o in formulation [1] were set at the solution levels for Y_1 and Y_2 in formulation [2], the minimum cost (budget) in formulation [1] would be equal to X_1^o in formulation [2], and the Z_1 and Z_2 in each solution would be the same.

Similarly, a set of cost-efficient alternatives could be generated with formulation [1] by repeated solution with parametric variation in Y_1^o and Y_2^o. Essentially the same thing could be accomplished with formulation [2] by parametrically varying X_1^o, P_1^o, and P_2^o. The two approaches are fundamentally equivalent.

Throughout this report, formulation [2] is generally used for two reasons. First, formulation [1] does not lend itself directly to the traditional formulation of a cost minimization problem, because one input (land) is not costed and is held fixed, which is rather unconventional. Formulation [2], in contrast, holds all inputs fixed, and maximizes some summation of outputs. A solution to formulation [2], therefore, can be interpreted as a point on a production frontier.

Second, formulation [2] allows prespecification of the budget level for the production frontier being analyzed. As will be shown, this is a desirable characteristic for the analysis to follow. Literally, the P_1 and P_2 will be varied to "sample" different points on a given production frontier.

This approach [2] also implicitly maximizes net revenues, because, with a fixed budget, deducting costs from revenues in the objective function is redundant. The scarcity of the inputs is accounted for by the (presumably binding) constraint. For example, to maximize net revenues (or profit)

$$\text{Maximize: } P_1Y_1 + P_2Y_2 - (r_1Z_1 + r_2Z_2)$$

$$\text{Subject to: } f(Y_1, Y_2, Z_1, Z_2, X_2) = 0$$

and, subject to a binding budget and a binding land constraint,

$$r_1Z_1 + r_2Z_2 = X_1^o$$

$$X_2 = X_2^o.$$

This problem can then be rewritten as

$$\text{Maximize: } P_1Y_1 + P_2Y_2 - X_1^o$$

$$\text{Subject to: } f(Y_1, Y_2, X_1^o, X_2^o) = 0.$$

Because X_1^o is a constant ($r_1Z_1 + r_2Z_2 = X_1^o$), its presence in the objective function does not influence the solution (assuming second order conditions are met) but does influence the shadow prices (on X_1^o and X_2^o). Thus, with a fixed budget, the only motivation for choosing this formulation rather than [2] is whether shadow prices are desired in terms of net revenues or gross revenues. It will be useful here to use formulation [2] so that the budget is fully utilized, even if it implies a solution with less than maximum net revenues. That is, with formulation [2], only the relative magnitude of the output prices to themselves is important; the scale of output prices relative to cost levels is irrelevant. Hereafter, the Z's will be suppressed, and X_1 will be used for simplicity. Note, however, that determination of the vector of inputs (Z_i) purchased with the budget (X_1) is an important part of solving formulation [2]. In the linear programming formulations discussed later, it is principally by varying land allocation and by varying this Z vector that different output vectors are generated. In these linear programs, there are rows to represent budgetary inputs (X_1) and land inputs (X_2), and the Z vector varies across columns that are "management prescriptions" applied to the land.

In national renewable resource planning, an additional problem arises. Whereas the total amount of the land inputs as well as its allocation across local planning units is fixed (by geographic boundaries), even if the total budget input is fixed, the allocation of budget across local planning units is not fixed. And, the allocation of total outputs to local planning units also must be determined (along with total output levels in the first place). Thus, the national problem can be formulated as

$$\text{Maximize: } P_1Y_1 + P_2Y_2$$

$$\text{Subject to: } f_i(X_{1,i}, X_{2,i}, Y_{1,i}, Y_{2,i}) = 0 \;;\quad i=1,n$$

$$\sum_{i=1}^{n} X_{1,i} \leq X_1^o$$

$$X_{2,i} = X_{2,i}^o \;;\; i = 1,n$$

$$\sum_{i=1}^{n} Y_{1,i} = Y_1$$

$$\sum_{i=1}^{n} Y_{2,i} = Y_2$$

or, more simply

$$\text{Maximize: } P_1(\textstyle\sum_i Y_{1,i}) + P_2(\textstyle\sum_i Y_{2,i})$$

$$\text{Subject to: } f_i(X_{1,i}, X_{2,i}^o, Y_{1,i}, Y_{2,i}) = 0 \;;\; i=1,n$$

$$\sum_{i=1}^{n} X_{1,i} \leq X_1^o \qquad [3]$$

where the i subscript indicates the i^{th} local planning unit's "share" of the given variable, and n indicates the number of local planning units.

If it were not for constraint [3], that is, if the allocation of budget to local units was predetermined, or if there was no global budget constraint (only local ones), then the global solution would simply be the collection of local solutions. That is, collect, for all i=1,n

$$\text{Maximize: } P_1Y_{1,i} + P_2Y_{2,i}$$

$$\text{Subject to: } f_i(X_{1,i}^o, X_2^o, Y_{1,i}, Y_{2,i}) = 0.$$

To use constraint [3] as the only reason for national planning is to take a very limited view of the reasons for national planning. However, it provides a workable formulation for testing multilevel optimization.

Linear Programming Analysis of Managed Forest Ecosystems

Renewable resource management and planning problems often have been analyzed using mathematical programming techniques such as linear programming (D'Aquino 1974, Ashton et al. 1980). The basic structure of the linear programs commonly used to analyze managed renewable resource ecosystems is depicted in table 1. The simplistic example in table 1 ignores time dimensions and constraints such as budget limitations, even flow restrictions, and minimum output levels. These complexities do not pose analytical problems, but do make the models quite large in some cases. Also, for simplicity, table 1 only includes two outputs: timber and forage.

In table 1, the major column headings are types of land and/or resources. The C_1 through C_5 columns are the number of acres allocated to alternative management prescriptions which could be applied in TYPE 1 (C_1 and C_2) and TYPE II (C_3, C_4, C_5) land. The Timber and Forage rows in the A matrix represent the resource flows per acre that result from implementation of the management prescriptions. For example, $A_{1,1}$ is the output of

Table 1.—A simple depiction of typical linear programs used in renewable resource management and planning.

	Type I		Type II			Outputs		Type	Right-hand side
	C_1	C_2	C_3	C_4	C_5	Timber	Forage		
Timber	$A_{1,1}$	$A_{1,2}$	$A_{1,3}$	$A_{1,4}$	$A_{1,5}$	-1		=	$K_1 = 0$
Forage	$A_{2,1}$	$A_{2,2}$	$A_{2,3}$	$A_{2,4}$	$A_{2,5}$		-1	=	$K_2 = 0$
TYPE I	1	1						=	K_3
TYPE II			1	1	1			=	K_4
Objective function	$-A_{5,1}$	$-A_{5,2}$	$-A_{5,3}$	$-A_{5,4}$	$-A_{5,5}$	$A_{5,6}$	$A_{5,7}$		

timber for each acre on which management prescription C_1 is implemented. The TYPE I and TYPE II rows are the land inputs to this production system. K_3 acres of Type I land are available, and K_4 acres of Type II land are available.

The Outputs (Timber and Forage) are accounting columns that collect the outputs described in the first two rows into aggregate outputs for the area being analyzed. K_1 and K_2 are set at zero to force all product output levels into these columns. The coefficients in row 5, the Objective Function row, describe the change in net revenues if one unit of C_1 occurs. Thus, $A_{5,1}$ is the cost of prescription C_1 on one acre, and $A_{5,6}$ is the revenue derived from one unit of timber output. This row is often the objective function to be maximized. As discussed previously, cost (budget) might also be constrained and revenue might be maximized, or outputs might be constrained and cost might be minimized.

Problems in Applying Linear Programming at the National Level

Planning for optimum management of national renewable resources is a staggering problem because of conflicting needs for detail and scope. Analyzing relatively small areas of land (such as a National Forest) is appealing because of the relative detail, resolution, and accuracy that can be achieved in a model such as that depicted in table 1. However, common inputs or outputs may enter the problem across local boundaries, increasing the desirability of a larger scale analysis that can capture absolute and comparative advantages between local land units. Thus, local level optima cannot simply be added up to yield an optimal national plan. The ideal solution would be the use of a single national analysis that is capable of achieving high levels of resolution and detail. Unfortunately, such a model would be huge and unworkable. For example, typical FORPLAN (Johnson et al. 1980)[3] models used at the national forest level in USDA Forest Service land management planning are large enough to stress the capabilities of modern computer systems. Attempting to agglomerate the FORPLAN models from the 120 forest planning units into one national model would not be possible.

[3]*Johnson, K. N., D. B. Jones, and B. M. Kent. 1980. Forest planning model (FORPLAN) user's guide and operations manual. Available from Systems Application Unit, Land Management Planning, USDA Forest Service, Fort Collins, Colo.*

Decomposition Models

One approach to solving certain very large optimization problems is "decomposition" of the problem. As Dantzig and Wolfe (1961) state:

> Many linear programming problems of practical interest have the property that they may be described, in part, as composed of separate linear programming problems tied together by a number of constraints considerably smaller than the total number imposed on the problem.... It would seem that each of the *n* sets of constraints ... constitutes a "subproblem" of secondary importance to the whole program, and that they should be studied mainly through the restrictions they impose on the activities of the "joint" constraints...

Dantzig (1963) noted that,

> The price paid for this decomposition is that the master program and the subprogram may have to be solved several times. First the master program is solved, and from its solution, objective functions are generated for each of the subprograms. Then these are solved, and from their solution new columns are generated to be added to the master program. The process is then repeated until, after a finite number of cycles, an optimality test is passed.

Kornai and Liptak (1965) emphasized the interpretation of decomposition as a multilevel optimization problem. Kornai (1975) later clarified that the real distinction between Dantzig and Wolfe's approach and theirs was that theirs was a "direct" procedure that did not go through the so-called "extremal problem." Kornai (1975) also clarified that both approaches (and a number of other ones) can be considered to be members of the "family of decomposition procedures."

The Kornai and Liptak (1965) approach (for a two-level problem) involves a "game-theoretical model" between a higher level planning authority (the "center") and a set of sectoral planning units. The center makes an initial, provisional distribution of the "available resources, material, manpower, etc. among the sectors, and at the same time also indicates their output targets." The sectors then rigorously analyze this set of "quotas" and report back "one type of economic efficiency index—the shadow prices derived from programming." The center then modifies the resource and output "quotas" based on this information. By iterating back and forth, a sectoral allocation is arrived at that, within a given tolerance level, equates the shadow prices across sectors, and thereby reaches a global optimum.

Both the Dantzig-Wolfe (DW) approach and the Kornai-Liptak fictitious play (FP) approach are shown to converge to a solution with continued iterations between sectors and the center. In further comparing the two basic approaches, Kornai (1975) stated:

> ...two main classes may be distinguished. In one of these shadow prices are sent down from above and indicators of the volume type are sent upwards from below. The "ancestor" of this class is the DW algorithm. In the other class, volume type indices are sent down from above and shadow prices are sent upwards from below. The pioneer of this second class was the FP algorithm.

Problems in Applying a Decomposition Model at the National Level

There are two principal problems in applying a DW or FP model to a national renewable resource planning problem. First, it would be quite rare for all of the local planning units (such as national forests) to complete their planning efforts simultaneously. Second, the communications network and coordinating authority to implement the repeated iterations necessary in a DW or FP model generally are not present. An alternative is needed that allows the local planning units to operate more independently and still allows an integration of these efforts without serious suboptimization (in the global sense).

The Bartlett-Wong Approach

Work by a number of researchers, most notably Bartlett (1974) and Wong (1980), suggests that national level analyses could focus on control by the selection of discrete management alternatives provided (perhaps over a period of years) by the lower levels of the organization. This approach has not been developed or tested. This report develops the details of this approach and tests it in a case example.

Figure 1 depicts the basic structure of this analytical approach. The local level linear programs are used to construct discrete management alternatives for consideration at the national level. The national level model would also most logically be a mathematical program. Table 2 depicts the programming matrix of a very simple linear programming version of a national model. In this example, only two local planning units, two alternatives, and two products are included. Also, only one time period is included. Expansion beyond the dimensions of this simple example is relatively straightforward.

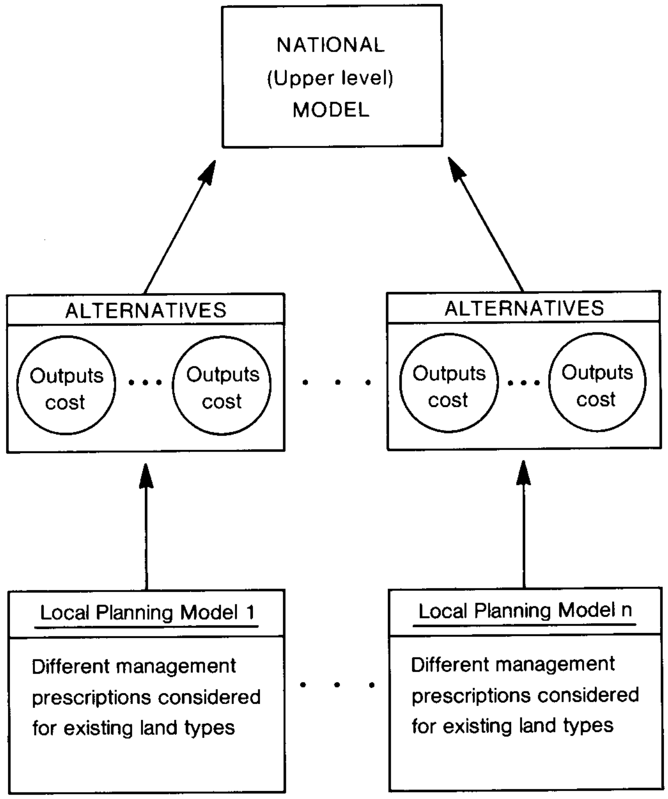

Figure 1.—Multilevel modeling structure for national level planning analysis.

In table 2, X_1^1 through X_2^2 are 0–1 variables representing selection or rejection of an alternative output vector with associated joint cost ($A_{3,i}$; i = 1,4), for a given local planning unit. For example, X_1^1 represents selection (1) or rejection (0) of the output vector $A_{1,1}$ and $A_{2,1}$ in Local Planning Unit 1. The first two rows collect the outputs from selected alternatives, and the fifth row is the objective function to be maximized. In this case, (global) budget is constrained (fourth row) and revenue is maximized. All of the matrix below the objective function row constrains the X_1^1 through X_2^2 so that each of them is between 0 and 1, and so that a "total" of only one alternative can be selected for each local planning unit. Because this is a linear programming model with continuous variables, X_1^1 through X_2^2 may actually take on solution values between 0 and 1 but not equal to either. For example, X_1^1 and X_2^1 in table 2 might solve with values of 0.6 and 0.4, respectively. This is interpreted as a partial selection of each alternative, the combination of which satisfies the "0–1 model constraints." This can be avoided if the national model is solved with an integer program. This option is discussed later.

The principal advantage of this multilevel optimization is that the detail and high resolution of local level analyses are preserved, but national optimization with a global budget constraint is still allowed—the national optimum will not simply be a summation of local optima. The implied national model reflects much detailed production analysis, but is itself of very workable size and complexity. And, any national model solution can automatically be disaggregated to a set of local management

Table 2.—A simple upper level model structure.

	Local planning unit 1		Local planning unit 2		Outputs		Cost	Type	Right-hand side
	X_1^1	X_2^1	X_1^2	X_2^2	Timber	Forage			
Timber	$A_{1,1}$	$A_{1,2}$	$A_{1,3}$	$A_{1,4}$	-1			=	0
Forage	$A_{2,1}$	$A_{2,2}$	$A_{2,3}$	$A_{2,4}$		-1		=	0
Cost	$A_{3,1}$	$A_{3,2}$	$A_{3,3}$	$A_{3,4}$			-1	=	0
Global budget constraint							1	≦	K
Objective function					B_T	B_F			MAX
0-1	1	1						=	1
Model constraints			1	1				=	1

plans. The main shortcoming of this multilevel optimization approach is that limiting the national analysis to a finite, in fact relatively small, number of discrete choices may overlook desirable options and lead to suboptimization.

The Bartlett-Wong approach might be contrasted with the DW and FP approaches as follows. In the DW and FP approaches, the center (national planners) and the sectors (local planning units) iterate back and forth to converge on a solution that is known to be optimal within some prespecified level of tolerance. In the process of this iteration, the sectors must run their local models several (perhaps many) times, and each run is directed by the center based on the information gained from the previous iteration(s). In the Bartlett-Wong model, however, the center must direct the sectors to perform a full set of runs based only on whatever information is available. These local runs may be performed at different times, as the models are available. The center must try to anticipate a set of sectoral runs that will be needed in searching for a global optimum (or an approximation thereof). The DW and FP approaches were shown (Dantzig and Wolfe 1961, Kornai and Liptak 1965) to always converge to approximate optimal global solutions. Because such a proof is not possible for the Bartlett-Wong approach, a test case is presented here to evaluate its tendency to suboptimize. The Bartlett-Wong approach is considered as a possibility because it currently seems pragmatically more useful than the DW and FP approaches for national renewable resource agencies. In any situation where the more elegant DW or FP type of approach is pragmatically feasible, its use probably would be preferable, because each has been shown to always converge to a given approximation of optimality. The test case evaluation of the Bartlett-Wong model cannot be used to generalize, and should be interpreted accordingly. All inferences and conclusions drawn are based solely on this single experiment.

Alternative Forms of the Bartlett-Wong Approach

Different Ways to Derive Lower Level Alternatives

Figure 2 is a simple production possibilities curve for a given lower level planning unit, one time period, and two outputs. Timber and forage are competitive outputs with one product transformation relationship, described by arc AB. This particular product transformation curve corresponds to an implicit budget level b_1. Larger budgetary expenditures, such as b_2, would shift the transformation curve to the right, as in arc CD. An alternative describes some mix of timber and forage outputs, and is depicted by a point, such as P_1, on one of the tranformation curves. The set of alternatives used to describe this simple production unit's capabilities should approximate the product transformation relationships at various budget levels. For example, alternatives P_1, P_2, and P_3 might approximate these relationships for budget b_1, while P_4, P_5, and P_6 correspond to budget b_2, and so forth.

Given such a production unit, systematic development of alternatives could proceed from at least three different approaches:[4]

1. Set the budget at a particular level, for example b_1, and obtain alternatives (e.g., P_1, P_2, and P_3) by varying the *relative* output prices (i.e., the objective function arguments) and maximizing revenue. This could be done for several budget levels. This would result in a group of alternatives depicted by the P's in Figure 2.
2. Set the output mix (e.g., at A_2) and minimize costs to determine the required budget level. This could be done for a number of output mixes. Production possibility curves could only be approximated if, by chance, alternatives with similar budget requirements obtained. This approach generally would result in a group of alternatives depicted by the A's in Figure 2.
3. Set the absolute output prices at a variety of levels and then maximize net revenue with no budget constraints. Different budget levels and product mixes would result from the different absolute magnitude of the prices. This approach also would generally result in a group of alternatives depicted by the A's in Figure 2.

[4] In current practice, alternatives often are not developed in a systematic manner (i.e., they are developed with a combination of the three different approaches discussed here). This probably occurs because local planning efforts are directly oriented towards local planning. If local planning efforts are to support a multilevel national planning analysis, then systematic development of alternatives would be especially desirable.

Approaches 2 and 3 generally will not yield groups of alternatives that can be associated with a particular budget. It would only be chance that more than one alternative would have the same budget level. For the analysis discussed here, it was judged to be very desirable to have alternatives placed into groups with the same budgets.

Briefly, this type of grouping produces a model where successive (each associated with a given budget) production possibilities curves are piecewise approximated. Approaches 2 and 3 yield a scattering of points, each on a different production possibilities curve. Interpolations between alternatives that have the same budget level are viewed differently than interpolations between alternatives that have different budget levels. Only approach 1 allows making this distinction.

Another important consideration is that the global budget will be the constraint that cuts across all local planning units and thus makes the national model necessary. Approach 1 is the only one (including combinations of the approaches) that allows the analyst to ensure a particular level of variation in local budgets across the local alternatives developed. Budgetary variation in the alternatives is critical to the performance of the multilevel model. Using either approaches 2 or 3 might result in only very narrow ranges in local budgets or ridiculously wide ranges in local budgets that leave too much unknown space between them. Also, with approach 3, unless considerable information on costs and output prices is available, meaningless results may be obtained because of absolute prices being set way out of scale with costs. Finally, approach 1 is clearly advantageous in terms of model interpretation.

Approach 1 therefore, will be adopted for the remainder of this discussion. Within this approach, significant questions remain involving the number of alternatives that are desirable and the specific definition of the different alternatives included. These questions are examined in the test case.

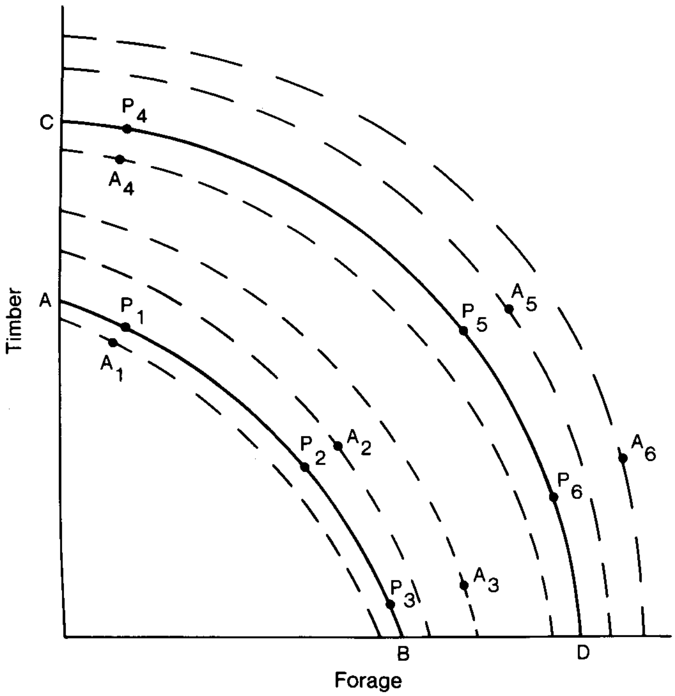

Figure 2.—A two-output, one-time-period, production possibilities curve for a given local planning unit.

Linear Programming Versus Integer Programming Solutions to the Upper Level Model

Referring to table 2, the point was made that if the upper level model is solved with a linear program, X_1 through X_4 may actually take on solution values between 0 and 1. For example, X_1 and X_2 in table 2 might solve with values of 0.6 and 0.4, respectively. Again, this is interpreted as a partial selection of each alternative, the combination of which satisfies the "0–1 model constraints." It is basically a linear interpolation between two alternatives. The interpolated cost would be only an approximation of unknown reliability (investigated below). An alternative approach would be to solve the upper level model in a way such that only complete alternatives can be discretely selected.

Three options should be considered. First is the completely continuous variable linear program as depicted in table 2. Second is the option of treating all alternatives discretely and solving the model with an integer program where all choice variables are constrained to be zero or one. This model would look exactly like that in table 2, except that X_1 through X_4 would be constrained to be integers. The third option would allow interpolations between alternatives that have the same budget, but would not allow interpolations between alternatives that have different budgets. This distinction is important, because interpolations between alternatives with the same budget merely create a piecewise approximation of a given production possibilities curve. An interpolation between alternatives with different budgets involves approximating the rate of expansion of the product transformation curve (with increasing outlay). The first type of interpolation is different from the second from an economic point of view, and the former may be more tenable (safer) than the latter. Table 3 depicts a simple model of this type. It is similar to that in table 2, but includes four alternatives (X_1 – X_4) for each local planning unit. It also includes rows that create the "mixed integer" model.

Figure 3 depicts the production possibilities map for a given local planning unit (where a partial selection occurs) with the first option—a continuous variable linear program. All points between any two alternatives are allowed (potentially) in the solution. In contrast, figure 4 depicts a similar production possibilities map that results from the second option—an all-integer program that treats all alternatives as discrete choices. Only the discrete alternatives specified in the upper level model are allowed. The third option is similarly depicted in figure 5. As indicated previously, this is a compromise that allows all points between alternatives with the same budget but does not allow points between alternatives with different budget levels to enter solution.

With only one global (budget) constraint and linear objective functions, the distinction between option two and

Table 3.—A simple mixed integer upper level model structure.[1]

	Local planning unit 1				Local planning unit 2				Integer variables				Outputs		Cost		
	Budgets				Budgets												
	1		2		1		2										Right-hand
	X_1^1	X_2^1	X_3^1	X_4^1	X_1^2	X_2^2	X_3^2	X_4^2	I_1^1	I_2^1	I_1^2	I_2^2	Timber	Forage		Type	side
Timber	t	t	t	t	t	t	t	t					-1			=	0
Forage	f	f	f	f	f	f	f	f						-1		=	0
Cost (Budget)	c	c	c	c	c	c	c	c							-1	=	0
Mixed integer constraints	1	1							-1							=	0
			1	1						-1						=	0
					1	1					-1					=	0
							1	1				-1				=	0
									1	1						=	1
											1	1				=	1
Objective function													B_T	B_F			MAX
Global budget constraint															1	≤	K

[1]*To conserve space, the subscripted A-matrix (as in table 2) is represented here simply with the "t," "f," and "c" entries.*

option three is moot. That is, under these conditions, option three will always result in an all-integer solution — the solution would always be one of the corner points in figure 5. However, using option three may provide for considerable computational savings and improved solvability, relative to option two. Because the tenability and usability of options one and three are not yet clear, they are discussed in the test case also.

Number of Levels

Another issue concerning the configuration of a multilevel optimization model involves the number of levels in the model. In a given institutional setting there may be requirements for a particular number of levels. However, if the objective is to approximate a national optimum, only two levels should be employed.

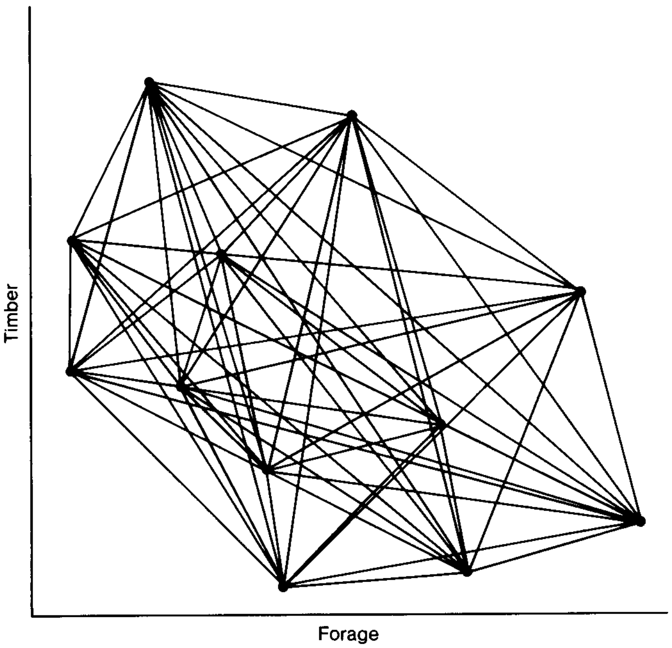

Figure 3.—A graphical depiction of the production space for a given local planning unit in a continuous variable linear program upper level model.

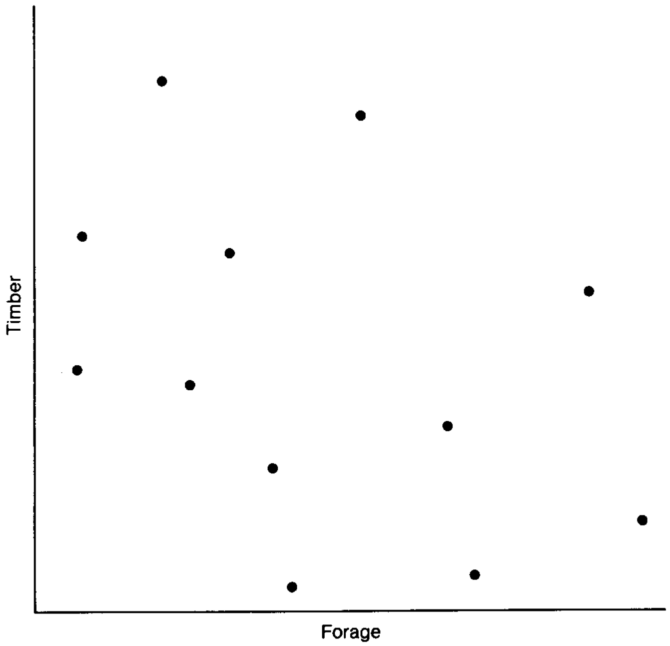

Figure 4.—A graphical depiction of the production space for a given local planning unit in an all-integer program upper level model.

As an example, consider a planning problem with 100 lower level planning units. Assume that there are only 10 discrete alternatives to be considered for each lower level model. With two levels, the higher level model for this problem in the form of table 2 would have 1,000 columns. Next consider the insertion of an intermediate level such that the same problem would be analyzed with three levels. That is, the lowest level would generate alternatives for the intermediate level model, and then the intermediate level model would generate alternatives for the national level model. Both the intermediate level models and the national model would be in the form of table 2. If each intermediate level planning unit is composed of 10 of the lowest level planning units, then each of the intermediate level models would have 100 columns. However, if all of the information (alternatives) in the two-level model is to be retained at the national level of the three-level system, then each intermediate planning unit would have to pass up 10^{10} discrete alternatives to the national model—every combination of lowest level alternatives. The national model would then have 10^{11} columns (10^{10} columns for each of 10 intermediate level planning units). If partial selections are allowed between the discrete alternatives, or if more alternatives per local planning unit are included, then the problem gets substantially worse.

If a particular institutional setting requires more than two levels, this must be weighed against the increased technical difficulties before adopting either approach. Again, it is assumed that the objective is to approximate national optima, not to integrate several levels of planning. The test case demonstrates that intermediate level information can easily be reported (or for that matter constrained) in the two-level approach.

In summary, both linear programming and integer programming versions of the upper level model in a two-level system should be tested for suboptimization and resource misallocation. Also, different numbers and types of local alternatives might be included in the upper level models, and these options should also be tested. These local alternatives will be generated by constraining local cost (budget) to a variety of levels and maximizing revenue with a variety of relative timber and forage output prices.

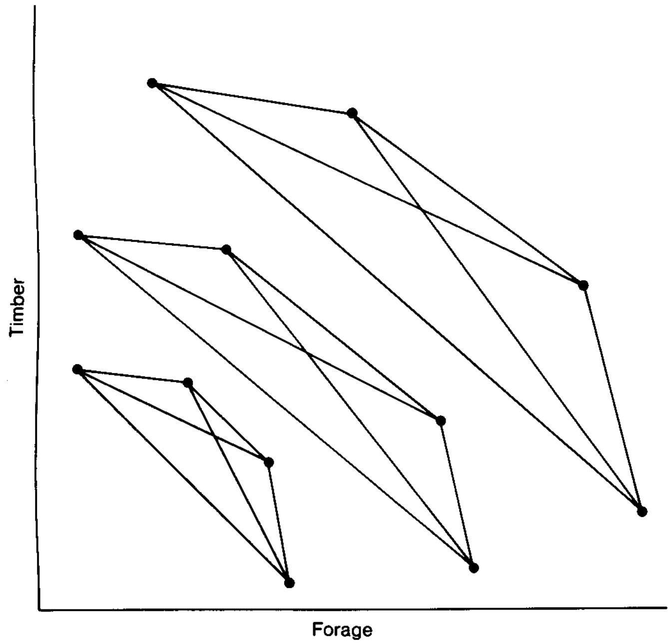

Figure 5.—A graphical depiction of the production space for a given local planning unit in a mixed-integer program upper level model.

A Test Case

The basic approach to the test case was to locate a suitable single-level global model that would serve as a standard for comparison, and then to build multilevel (two-level) models out of the global model. The lower level models in the multilevel system were built simply by subdividing the global model (Appendix 1) geographically. The alternatives for the upper level model(s) were generated by repeatedly optimizing the subdivisions (local planning units) with different local budget constraints and different relative price vectors in the revenue-maximizing objective function. Next, the upper level model(s) were tested for suboptimization with a variety of relative price vectors and global budget constraints. A mathematical description of the models used in the test case is presented in Appendix 2.

The Global Model

The model used for the test case was the National Interregional Multiresource Use Model (NIMRUM) for the land base described here. The NIMRUM model (Ashton et al. 1980) was developed for use by the USDA Forest Service to satisfy the requirements of the Forest and Rangeland Renewable Resources Planning Act of 1974 (RPA). A complete description is presented in Appendix 1. Its basic structure is similar to that depicted in table 1. The data were developed using an interdisciplinary team approach. The participants in the interdisciplinary teams were mostly Forest Service employees, together with some other federal agency and state government individuals.

The land base used in developing the test case models included all forest and range lands in USDA Forest Service Regions 4, 5, and 6, which includes all of or major portions of Idaho, Wyoming, Utah, Nevada, California, Oregon, and Washington. This large area was chosen to have a wide variety of production capabilities and ecosystem types. Because the context of the test case is a system for national planning analyses, this was considered to be an important test characteristic.

In all of the multilevel model tests, the local planning units are the same. There are 14 of them consisting of the 7 states divided into National Forest System and non-Forest System lands. This was merely a convenient way to define a reasonable number of local planning units. The analysis is organized simply to emulate a large-scale resource planning problem. No policy implications are intended regarding the actual production or cost results or regarding any apparent comparisons between different lands or ownerships.

Ten Different Upper Level Models

Two terms are defined for the discussion that follows. First, model "type" indicates whether the upper level model is a continuous variable linear program, or an integer program. Second, model "configuration" indicates the number and breadth of the different alternative output vectors included in the upper level model for each local planning unit.

Five different upper level model configurations were constructed, each with a different set of alternatives included for each local planning unit. Each configuration was constructed for the two solution types: continuous variable linear programming and integer programming. This yielded 10 different upper level models. As discussed previously, a mixed integer formulation that always yields an all-integer solution was used for computational efficiency.

Forage and stumpage prices vary significantly across the global model land base (Forest System Regions 4, 5, and 6). The NIMRUM model assumed different prices for each region. This assumption was adopted in this study also. This amounts to defining different timber and forage outputs for each region. In any actual national planning effort, commodities probably would not be homogeneous across the entire nation. Regional pricing provides a simple way of depicting this pragmatic complication. In an actual planning problem, a simple regional commodity definition may not be the best approach. The base prices, based on those used in USDA Forest Service (1980), are as follows.

	Timber $/1,000 ft³	Forage $/AUM
Region 4	303	4.96
Region 5	833	6.00
Region 6	1146	5.90

The relative price vectors used in building the multilevel model(s) are all multiples of the base prices, and thus retain the proportional regional price differences.

A pool of alternatives for each local planning unit was developed, from which the different upper level models could be built. First, a "base" run was generated for each local planning unit. These were unfettered (no budget constraint) local maximizations of net value, using the prices described. Based on the costs (budget outlays) for each local planning unit in these base runs, the following four budgets were defined for each local planning unit: (1) 50% of the local base budget; (2) 75% of the base budget; (3) 100% of the base budget; and (4) the local budget required when output prices were doubled and an unfettered net value optimization was run. The last option was included to permit a greater than base level production capability that is not arbitrarily high, but instead is associated with particular price increases.

After the four budget levels were defined for each local planning unit, sets of points on the production possibility frontiers were located by fixing the budget level and maximizing weighted outputs. In this context the weights can be viewed as relative prices. Five different relative price vectors were constructed. Because variation in relative prices is desired, the forage prices were held at their base levels. Each of the relative price vectors was run with each local budget level to generate a pool of 20 alternatives for each of the 14 lower level models.

Five different upper level model configurations were built from this pool of local alternatives, for which the following codes were used.

Local pricing code	Local relative price vectors
1	Stumpage prices at 10 times the base stumpage prices
2	Stumpage prices at 3 times the base stumpage prices
3	Stumpage prices at base stumpage prices
4	Stumpage prices at 1/3 of the base stumpage prices
5	Stumpage prices at 1/10 of the base stumpage prices

Local budget code	Local budget level
1	50% base
2	75% base
3	100% base
4	Twice the base price net value maximization budget

The five different upper level model configurations (labeled 1, 2A, 2B, 3, and 4) included the following alternatives for each local planning unit.

Configuration	Local budgets included	Local prices included	Total number of alternatives for each local planning unit
1	1,2,3,4	1,2,3,4,5	20
2A	1,3,4	1,3,5	9
2B	1,3,4	2,3,4	9
3	1,4	1,5	4
4	1,3,4	3	3

Nine Tests of the Upper Level Models

The global model and each of the 10 upper level models were solved with three different global budget constraints,[5] and three different relative price vectors for each global budget constraint. Comparing these solutions provides the tests for suboptimality in the upper level

[5]The term "global budget constraint" applies to a fixed budget level for the entire planning area (all 14 local planning units). This "global budget constraint" is applied to both the "global" model and the "multilevel" or "upper level" models.

models. The three global budgets for testing purposes were

Low: 0.625 times the sum of local base (code 3) budgets
Medium: 0.875 times the sum of local base (code 3) budgets
High: The midpoint between the sum of local base (code 3) budgets and the sum of code 4 local budgets.

The three different relative price vectors used in testing the upper level models were:

Low relative timber prices: 0.2 times the base timber prices
Medium relative timber prices: Base timber prices
High relative timber prices: 5 times the base timber prices.

All timber prices are taken with the base forage prices.

The partial selections that resulted from linear programming solution of the upper level models also were analyzed to determine the accuracy of the interpolated cost implied. All nine tests were performed for both linear programming solutions and integer programming solutions of the five different upper level model configurations. Thus, the global model was solved nine times, and each of the 10 upper level models was solved nine times with budgets and relative prices varied systematically. Comparing these solutions indicates the tendency of the upper level models to suboptimize.

Results and Discussion

Objective Function Test Results for Linear Programming Solution of the Upper Level Models

Table 4 presents a set of ratios between the five different upper level linear program solution objective functions and the same for the global model. This gives a general picture of the suboptimality introduced by the multilevel alternative to global optimization.

For the Model 1 configuration, the upper level model objective function solution value is clearly very close to the global one. The only instance where the suboptimality (as measured here) is more than 1% is in the test using the low budget and high relative timber prices. This test is stressing the multilevel model severely and is still within 4% of the global model. On the basis of these results, if a model configuration as large (20 alternatives per lower level planning unit) as Model 1 can be used, relatively reliable optimality results should be expected. That is, these results indicate minimal suboptimality for Model 1.

Models 2A and 2B (which each have nine alternatives for each lower level planning unit) do not perform much worse. The only instance where the suboptimality is more than about 2% is, again, the test with the low global budget constraint and the high relative timber price. In this instance, the suboptimality is about 5% for each model. Again, minimal suboptimality has occurred, based on these figures.

The fourth column—applying to Model 3—is less encouraging. This model had only four alternatives per local unit, and only two different local budget levels were included for each lower level land unit. This configuration resulted in suboptimization as much as 13.5% and only rarely resulted in suboptimization less than 5%.

In contrast, the last column—applying to Model 4—is more promising. This model had only three alternatives for each lower level land unit, and yet performed better than Model 3. The reason is that Model 4 included three different budget levels for each local planning unit. Because the global budget constraint creates the need for higher level planning and modeling in the first place, it is reasonable that more variation in the budget allocation across lower level land units would be more important than sheer number of lower level alternatives.

Other implications are evidenced by comparing the different columns in table 4. Models 2A and 2B both included nine alternatives for each local planning land unit, and the suboptimality results are essentially the same. Remembering that the only difference between Models 2A and 2B are the price vectors used in developing the local level alternatives, it is apparent again that budget variation across lower level alternatives is more important than price vector variation. This is equivalent to saying that for national optimization (as formulated here),

Table 4.—Ratios of objective function solution values of different upper level linear programming models to those of the global model.

Global budget constraint	Relative timber prices	Upper level linear programming				
		1	2A	2B	3	4
High	High	0.9908	0.9900	0.9869	0.9839	0.9723
High	Medium	.9942	.9942	.9942	.9276	.9942
High	Low	.9931	.9911	.9897	.9683	.9725
Medium	High	.9921	.9891	.9881	.9175	.9556
Medium	Medium	.9977	.9898	.9898	.8650	.9898
Medium	Low	.9963	.9918	.9925	.9498	.9633
Low	High	.9632	.9517	.9461	.9093	.8983
Low	Medium	.9922	.9826	.9826	.8850	.9826
Low	Low	.9960	.9928	.9936	.9063	.9415

it is more important for local level analyses to study the effects of varying budgets than to study alternative output mixes at a given budget. Even in tests with relative timber prices far different (by a factor of 5) than the single price vector used in developing Model 4, it performs reasonably well.

In comparing Models 2A and 3, note that Model 3 is the same as Model 2A, except that in Model 3 the alternatives for each local planning unit with the middle local budget levels and the middle relative price vectors were not included. Thus, Model 3 only includes the "outside" alternatives, leaving out the "middle" ones. This results in Model 3 performing particularly poorly for the medium global budget and the medium relative timber price tests. Because these tests are the least extreme, and perhaps the most clearly relevant ones, this is a serious deficiency. The implication is that leaving out central alternatives in developing a multilevel model is not advisable, unless only extreme solutions are desired—not a likely situation at all.

In comparing Models 2A and 4, note that Model 4 is the same as Model 2A, except that in Model 4 the alternatives for each local planning unit with the high and low relative timber prices are not included. This exclusion results in increased suboptimality, but not nearly as much as was encountered in Model 3. This corroborates the earlier conclusions from comparing Models 3 and 4 and from comparing Models 2A and 2B.

Comparing Model 2B with Models 3 and 4 yields similar conclusions. If a model such as 2B is not feasible, then reduction in size along the lines of Model 4 appears to be safer than along the lines of Model 3. Retention of "middle" lower level alternatives and variation in local budget levels appear to be the most important factors in designing the configuration of the multilevel model, based on these tests.

The relatively good performance of Model 4 is encouraging, especially considering that the test case only involves two outputs and one time period. From a linear programming perspective, increasing the number of time periods and increasing the number of outputs are equivalent complications—for example, tracking timber over five time periods is modeled by simply defining five outputs: timber in time period one, timber in time period two, etc. Thus, in a practical planning situation, the number of outputs is likely to be much larger than two because of both a more complicated production system and the inclusion of different time paths of production (scheduling). As the number of outputs increases, the number of lower level land unit alternatives that would have to be generated increases dramatically for a multilevel model such as Models 1 through 3. For example, to emulate Models 2A and 2B that include three (two-output) price vectors for a case with 15 outputs (perhaps three outputs over five time periods) would require taking $3^{(15-1)}$ or 4.8 million price vectors in combination with each local budget level. Model 4, however, can be emulated with one price vector taken in combination with each local budget level, regardless of the number of outputs or time periods.

Cost Analysis on Partial Selections

As established earlier, each solution to each multilevel model presented in table 4 involves one "partial selection" of two lower level alternatives, for a single local planning unit. A natural concern would be that the cost estimate for that particular lower level land unit would be inaccurate (overestimated)—it is a linear interpolation between the costs associated with the two lower level alternatives (Hof et al. 1985). Thus, for a chosen set of these partial selections, the actual cost of the interpolated output vector was determined by rerunning the lower level model (minimizing cost with output levels constrained to the appropriate levels). Even if the interpolated cost is nearly correct, if the discrete alternatives were implicitly evaluated and approved by the local planning unit officials, then the partial selection may still be undesirable or infeasible for reasons not captured in the model.

Table 5 presents the results of the cost analysis on the selected partial selections. For each upper level linear program configuration (1, 2A, 2B, 3, 4), two solutions and their ancillary partial selections were selected—one with a partial selection close to an even 50–50 split and one with a partial selection close to an 80–20 split. The first column in table 5 indicates the model configuration that the partial selections came from. The second column indicates the local planning unit where the upper level model chose to make the partial selection. The third column indicates the two local alternatives (defined by local budget level and price set) that were partially selected. The fourth column indicates the percent of each alternative that was partially selected. The fifth column indicates the estimate of cost for the partial selection that was calculated (in the upper level model solution) as a linear interpolation between the two given alternatives' costs. The sixth column presents the actual minimum cost obtained by rerunning the lower level model with a cost minimization objective function and with timber and forage constrained to the levels implied by the partial selection. The last column indicates the percent error in the interpolated cost estimate, relative to the actual minimum cost.

All of the interpolated cost estimates are relatively accurate (within 4%), except for the two partial selections tested from the Model 3 configuration. This is not a surprising result, given that the Model 3 configuration specified only two (extreme) budget levels for each lower level model. This characteristic caused the Model 3 configuration to perform poorly from the viewpoint of global optimization, and also appears to cause it to perform poorly from a viewpoint of the accuracy of the interpolated cost estimate implied in a partial selection. Despite this, it is encouraging that with upper level model configurations that include variation in local budget levels, the cost estimate implied by a partial selection may not be terribly inaccurate. Pragmatic considerations may still make a partial selection unusable; but based strictly on the cost analysis in table 5, it may not be thoroughly untenable.

Table 5.—Cost analysis on certain partial selections from the upper level linear programming solutions.

Model	Local planning unit	Alternatives in Partial Selection: Budget/Price set	Alternatives in Partial Selection: Percent in solution	Interpolated cost estimate	Actual minimum cost	Error in interpolated cost estimate
				----------- *dollars* -----------		*percent*
1	Oregon/National forest lands	4/4 3/5	85 15	333,679,987	323,891,044	3.02
		2/3 3/3	50 50	219,770,010	218,666,081	0.50
2A	Washington/National forest lands	3/3 1/3	87 13	99,280,024	97,614,012	1.71
		3/5 1/5	55 45	81,979,976	76,116,690	3.62
2B	Oregon/Other lands	3/3 4/3	89 11	302,479,996	299,427,812	1.02
		4/2 3/2	59 41	373,720,025	364,972,040	2.40
3	Oregon/National forest lands	4/1 1/1	73 27	287,469,036	259,139,480	10.93
		1/1 4/1	54 46	228,628,158	202,504,350	14.14
4	Oregon/National forest lands	1/3 3/3	87 13	142,469,000	138,589,650	2.80
		3/3 4/3	58 42	292,219,956	287,234,960	1.74

It was anticipated that the 80–20 partial selection cost estimates would be more accurate than the 50–50 counterparts, because the 80–20 ones are proportionately closer to a point with known cost. In table 5, this was not always the case. Comparing the two partial selections for the Model 1 configuration, for example, the cost error for the 85–15 partial selection is about six times that for the 50–50 partial selection. The apparent reason for this result is that the production function being modeled is not close to being homothetic, and does not exhibit constant returns to outlay. This would indicate that errors in interpolated cost estimates are not very systematic, and would be difficult to predict. Again, information concerning response to changing local budgets is very important. In the partial selections tested for Model configurations 2A, 2B, and 3, the budget codes involved in the partial selection for each local planning unit are the same, and as expected, the interpolated cost estimates for 80–20 partial selections are more accurate than for the 50–50 partial selections. In the partial selections tested for the Model 4 configuration, different budgets are involved, and the 80–20 partial selection cost estimate is, again, less accurate than the 50–50 counterpart.

Objective Function Test Results for Integer Programming Solution of the Upper Level Models

Table 6 presents a set of ratios similar to those in table 4, except that the numerators of these ratios are the objective function solution values arrived at through integer programming solution of the upper level model (so as to avoid the partial selection). The differences between these ratios and the equivalent ratios in table 4 are also given, in parentheses.

One simple but important conclusion can be derived from table 6. The only instance where a signficant loss in the objective function takes place because of avoiding the partial selection is in Integer Model 3, which did not perform particularly well in table 4 in the first place. Even for Model 3, the integer constraints never cause an increase in suboptimality (as measured here) of more than 5 percentage points. Otherwise, the integer solution approach does not significantly affect the conclusions drawn on table 4. The results in tables 4 and 6 are slightly conservative. Table 4 is slightly conservative because the interpolated cost estimates for the partial selection are always high. Table 6 is slightly conservative because the integer solutions yield a very small slack (never more than 0.5%) on the budget constraint.

Given the information in table 5, and the possibility that problems other than cost errors may be involved in trying to use the partial selections implied by the models in table 4, the integer solution approach seems to be supported in terms of overall suboptimality. Given the mixed integer model formulation discussed earlier, it also seems possible to solve relatively large upper level integer models with relatively few integer variables—so computational problems should not be terribly serious. However, the case example involves only one global constraint. In cases with more global constraints, the discrepancy between linear programming and integer

Table 6.—Ratios of objective function solution values of different upper level integer programming models to those of the global model. The numbers in parentheses are the differences between these ratios and the ratios in table 4.

Global budget constraint	Relative timber prices	Upper level integer programming model				
		1	2A	2B	3	4
High	High	0.9900 (.0008)	0.9885 (.0015)	0.9846 (.0023)	0.9816 (.0023)	0.9708 (.0015)
High	Medium	.9939 (.0003)	.9939 (.0003)	.9936 (.0006)	.9273 (.0003)	.9936 (.0006)
High	Low	.9924 (.0007)	.9883 (.0028)	.9883 (.0014)	.9656 (.0027)	.9711 (.0014)
Medium	High	.9916 (.0005)	.9872 (.0019)	.9863 (.0018)	.9056 (.0019)	.9546 (.0010)
Medium	Medium	.9966 (.0011)	.9894 (.0004)	.9894 (.0004)	.8608 (.0042)	.9894 (.0004)
Medium	Low	.9963 (.0000)	.9895 (.0023)	.9895 (.0030)	.9453 (.0045)	.9625 (.0008)
Low	High	.9589 (.0043)	.9477 (.0040)	.9422 (.0039)	.8981 (.0112)	.8941 (.0042)
Low	Medium	.9904 (.0018)	.9775 (.0051)	.9775 (.0051)	.8666 (.0184)	.9775 (.0051)
Low	Low	.9952 (.0008)	.9928 (.0000)	.9928 (.0008)	.8598 (.0465)	.9415 (.0000)

programming solutions may be larger. This point also applies to all subsequent comparisons between linear programming solutions and integer programming solutions.

Total Output Test Results

Tables 7 and 8 present ratios between total timber and forage output solution values for the different upper level models and the equivalent solution values from the global model. The integer programming ratios are given in parentheses under the linear programming ratios in both tables. With several exceptions (especially the case of forage with the low global budget constraint and the high relative timber price), the global model's output mix is approximated relatively closely by both the linear and integer programming upper level models. The approximation is generally best with the medium relative timber price. This is not surprising, because the high and low relative timber prices are quite extreme and would thus tend to stress the model results significantly. With the low global budget constraint and high relative timber prices, forage is essentially no more than a residual output, which explains the highly unstable results under those conditions. The model configuration appears generally to affect the output mix only slightly, although Models 3 and 4 exhibit some occasional instabilities, especially in forage output levels. Also, it is very difficult to say that the linear programming models perform any differently, in general, than the integer programming counterparts.

The results presented in tables 7 and 8 are based on a rather gross aggregation of regional outputs. Because the regional outputs were priced differently to represent the qualitative and other differences between regional outputs, aggregating them as in tables 7 and 8 is technically inappropriate. Nonetheless, tables 7 and 8 suggest that within a reasonable range of relative prices, the multilevel modeling approach should perform rather well in approximating the global output mix that would be arrived at if a single-level global optimization were possible. This conclusion is especially tenable with moderately constraining global budget constraints and with relative price vectors not too far from those used in developing the multilevel model alternatives. With the more extreme relative prices tested, the results are far less promising.

Regional Output and Cost Allocation Test Results

The results from Models 2A and 4 were chosen for presentation to indicate the reasonable range of regional and lower level output solution results. The Model 1 configuration, with 20 alternatives per local planning unit, is probably too "optimistic" for any situation that involves more than two outputs and one time period.

Tables 9 and 10 present ratios between regional timber and forage output solution values, respectively, for the linear and integer (in parentheses) versions of Models 2A and 4 and the equivalent solution values from the global model. The regional timber and forage outputs are significant principally because these are at the highest

Table 7.—Ratios of total timber solution values of different upper level linear (integer) programming models to those of the global model.

Global budget constraint	Relative timber prices	Upper level linear programming				
		1	2A	2B	3	4
High	High	0.9776 (.9895)	0.9899 (.9957)	0.9422 (.9328)	0.9715 (.9465)	0.9129 (.9259)
High	Medium	.9754 (.9885)	.9754 (.9885)	.9754 (.9889)	.9349 (.9341)	.9754 (.9889)
High	Low	.9916 (.9942)	.9482 (.9283)	1.0496 (1.0332)	.8894 (.9035)	1.1040 (1.0912)
Medium	High	.9708 (.9645)	.9777 (.9636)	.9139 (.9093)	.8975 (.9677)	.8383 (.8419)
Medium	Medium	.9978 (1.0306)	1.0027 (.9923)	1.0027 (.9923)	.9262 (.9049)	1.0027 (.9923)
Medium	Low	.9603 (.9657)	.9008 (.9254)	1.0265 (1.0054)	.8406 (.8124)	1.1088 (1.1130)
Low	High	.9783 (1.0139)	.9755 (.9855)	.8986 (.9142)	.9404 (.9437)	.8049 (.8205)
Low	Medium	1.0127 (1.0268)	1.0089 (1.0515)	1.0089 (1.0515)	1.0007 (1.0656)	1.0089 (1.0515)
Low	Low	.9873 (.9882)	.9341 (.9315)	1.0330 (1.0338)	.8580 (1.0389)	.8949 (.8949)

Table 8.—Ratios of total forage solution values of different upper level linear (integer) programming models to those of the global model.

Global budget constraint	Relative timber prices	Upper level linear programming				
		1	2A	2B	3	4
High	High	0.9642 (1.0189)	0.6074 (.7797)	1.8045 (1.5083)	0.5018 (.7797)	2.0045 (2.1008)
High	Medium	1.0006 (.9982)	1.0006 (.9982)	1.0006 (.9970)	.7375 (.7399)	1.0006 (.9970)
High	Low	.9907 (.9907)	1.0104 (1.0159)	.9649 (.9638)	1.0230 (1.0016)	.9248 (.9227)
Medium	High	.9742 (1.0204)	.4314 (.5739)	2.3944 (2.3877)	.4963 (.8656)	3.1418 (3.1750)
Medium	Medium	1.0178 (1.0271)	1.0357 (1.0338)	1.0357 (1.0338)	.7503 (.7927)	1.0357 (1.0338)
Medium	Low	1.0022 (.9995)	1.0157 (1.0147)	.9766 (.9718)	1.0071 (.9929)	.9240 (.9234)
Low	High	2.6687 (2.7358)	2.1025 (2.6137)	11.7926 (11.9777)	2.1589 (2.4527)	16.4095 (16.6242)
Low	Medium	.9897 (.9533)	.9994 (1.0073)	.9994 (1.0073)	.7949 (.8131)	.9994 (1.0073)
Low	Low	.9919 (.9951)	1.0038 (1.0032)	.9849 (.9838)	.9094 (.8512)	.8949 (.8949)

Table 9.—Ratios of regional timber output solution values in upper level linear (integer) programming Models 2A and 4 to those of the global model.[1]

Global budget constraint	Relative timber price	Model 2A Region 4	Model 2A Region 5	Model 2A Region 6	Model 4 Region 4	Model 4 Region 5	Model 4 Region 6
High	High	0.9778 (1.0343)	0.7417 (.7417)	1.0304 (1.0185)	0.9084 (1.0044)	0.0925 (.0925)	1.0331 (1.0185)
High	Medium	.9259 (.9976)	.4927 (.4927)	1.0132 (1.0066)	.9259 (.9990)	.4927 (.4927)	1.0132 (1.0066)
High	Low	1.0224 (1.0403)	.5122 (.1867)	.9578 (.9487)	1.2520 (1.1771)	.3149 (.3149)	1.1167 (1.1216)
Medium	High	.8538 (.8045)	1.9292 (1.9292)	.9934 (.9934)	.6071 (.6354)	.1772 (0.0000)	.9584 (.9584)
Medium	Medium	1.1722 (1.1116)	1.1785 (1.1785)	.9442 (.9494)	1.1722 (1.1116)	1.1785 (1.1785)	.9442 (.9494)
Medium	Low	.8933 (1.0307)	.7401 (.7401)	.9084 (.8837)	1.1141 (1.1565)	1.2487 (1.2487)	1.1022 (1.0907)
Low	High	1.0405 (1.1236)	2.8935 (2.8935)	.8975 (.8829)	.7660 (.8737)	0.0000 (0.0000)	.8416 (.8263)
Low	Medium	1.1250 (1.4165)	∞ (∞)	.9309 (.8892)	1.1250 (1.4165)	∞ (∞)	.9309 (.8892)
Low	Low	.9173 (.9101)	∞ (∞)	.9145 (.9145)	1.1537 (1.1578)	∞ (∞)	1.1633 (1.1615)

[1] *The ∞ symbol indicates that no production of timber occurred in the global model solution for that region.*

Table 10.—Ratios of regional forage output solution values in upper level linear (integer) programming Models 2A and 4 to those of the global model.

Global budget constraint	Relative timber price	Model 2A Region 4	Model 2A Region 5	Model 2A Region 6	Model 4 Region 4	Model 4 Region 5	Model 4 Region 6
High	High	0.7778 (1.1452)	0.07068 (.07068)	0.5591 (.4483)	1.3005 (1.4741)	5.6815 (5.6815)	1.4573 (1.4579)
High	Medium	.9922 (.9834)	1.0022 (1.0022)	1.0099 (1.0153)	.9922 (.9789)	1.0022 (1.0022)	1.0099 (1.0153)
High	Low	1.0029 (1.0030)	1.0011 (1.0014)	1.0420 (1.0663)	.9478 (.9416)	.9897 (.9897)	.7544 (.7541)
Medium	High	.9577 (1.3383)	.06742 (.06742)	.1309 (.1309)	2.5662 (2.7381)	9.9011 (9.6907)	1.6012 (1.6012)
Medium	Medium	1.0267 (1.0212)	.9985 (.9985)	1.1734 (1.1731)	1.0267 (1.0212)	.9985 (.9985)	1.1734 (1.1731)
Medium	Low	.9993 (.9983)	1.0001 (1.0001)	1.0706 (1.0681)	.9449 (.9442)	.9884 (.9884)	.7624 (.7628)
Low	High	11.0774 (14.1740)	.4235 (.4235)	.2660 (.2519)	29.6089 (30.7981)	60.8789 (60.8789)	5.8909 (5.9172)
Low	Medium	1.0365 (1.0769)	.9978 (.9978)	.9389 (.9156)	1.0365 (1.0769)	.9978 (.9978)	.9389 (.9156)
Low	Low	.9877 (.9863)	1.0025 (1.0025)	1.0283 (1.0283)	.9349 (.9353)	.9907 (.9907)	.6555 (.6548)

level of aggregation that is clearly tenable. And, the regional output reallocation implied by the multilevel solutions (relative to the global solution) is actually very similar to the local planning unit reallocations. In comparing the multilevel output solution values with the global output solution values, the local planning units within each region tend to "move together."

In studying tables 9 and 10, it is clear that definite instabilities occur in the multilevel solutions regarding regional output mixes. When the medium relative timber prices are used, the multilevel solution emulates the global solution rather closely. In other cases, however, the regional output levels arrived at by the multilevel models vary from their counterparts in the global model by factors as large as 60. Because, in developing the multilevel models, a price vector was used that was identical to the "medium" price vector used in these tests, it can be concluded that the multilevel model will only approximate the global optimal regional output levels with price vectors close to those used in developing the multilevel model. In tables 9 and 10, the results are most unstable with the extreme budget constraints taken in combination with the extreme relative timber prices; therefore, it appears that there is an interaction between these two factors. Model 2A does not perform much differently than Model 4 in tables 9 and 10. Even Model 1 does not perform much differently. For example, the Model 1 equivalents to the first rows in tables 9 and 10 are as follows.

	Region 4	Region 5	Region 6
Table 9	0.9473	0.74170	1.0228
Table 10	1.1566	0.07068	1.0576

Again, it does not seem to make very much difference whether the multilevel model is solved in a linear programming or integer programming format—each version performs slightly better sometimes and slightly worse other times. The basic conclusion appears to be that unless relative price vectors used in the upper level optimization are close to one of the price vectors used in building the upper level model, the multilevel solution regional outputs may be significantly different from the true global optimal figures. With anything other than a moderately constraining global budget constraint, this instability is potentially serious, even with the least stressing price vectors. It is important, however, that the multilevel model is capable of emulating the global model in at least the least stressing circumstances. In many planning problems, the price vectors that are to be used are known ahead of time, and only moderately constraining global budget constraints (relative to an unfettered maximization of net worth) are relevant.

Table 11 presents ratios between the regional budget allocations from the linear and integer (in parentheses) versions of Models 2A and 4 and the equivalent solution values from the global model. These are reported simply for comparison with the results in tables 9 and 10. The regional budget allocations in the multilevel models are somewhat closer to their counterparts in the global model than are the associated output results. Runset 2A and 4 perform more or less similarly, and, again, the integer programming solution only performs worse than the linear programming approach in a few cases. With the high relative timber prices, the budget allocations became more unstable in the multilevel model. Overall, the results in table 11 appear to have averaged out some of the more severe results in tables 9 and 10, but basically demonstrate similar problems in the multilevel models, especially with the more extreme price sets.

Local Planning Unit Output and Cost Allocation Test Results

Table 12 presents ratios between local planning unit timber and forage output solution values for upper level Models 2A and 4 and the equivalent solution values from the global model. Because the results in table 12 are from tests with the medium relative timber prices and the medium global budget constraint, they represent an optimistic situation. With medium relative timber prices and the medium global budget constraint, the solutions of Models 2A and 4 are identical. Even in this optimistic case, the multilevel models can indicate local level output solution values as much as 250% different from the global solution values. As before, the integer programming format generally does not yield results that are much different from the linear programming format. An important exception follows.

Most of the deviation between the global timber output solution values and those from the multilevel model solutions seems to be concentrated in one local planning unit—Oregon "other" lands. That is, the timber outputs on all other planning units are greater than those in the global model, whereas timber is "underproduced" on the Oregon "other" lands. With the integer programming solution, this also happens in the Wyoming NFS lands, and the overproduction on Wyoming "other" lands is exacerbated. It is clear that any solution to the multilevel model would have to be subject to some careful examination by the local level planners, and no solution should ever be taken as absolute. Such post-solution evaluation and adjustment would actually be required no matter what sort of analysis was undertaken.

Table 13 presents ratios of cost solution values for the same set of tests as in table 12. Again, even in this optimistic case (medium global budget constraint, medium relative timber prices), it would appear that the multilevel model may result in a budget allocation across local planning units that is significantly different from that which would be arrived at through a global model. Again, the "error" seems to be concentrated in much the same way that it was in table 12 for timber outputs. With the integer programming solution approach, the local budgets chosen are different in two local planning units besides the one where the partial selection occurred in the linear programming format. This results in a radically different budget for these local planning units, especially the Wyoming NFS planning unit. It would appear that the linear programming upper level model is significantly more stable than the integer programming approach in terms of local planning unit budget allocations. To a

Table 11.—Ratios of regional cost allocations in upper level linear (integer) programming Models 2A and 4 to those of the global model.

Global budget allocation	Relative timber price	Model 2A			Model 4		
		Region 4	Region 5	Region 6	Region 4	Region 5	Region 6
High	High	0.9325 (1.1600)	0.6890 (.6890)	1.0682 (1.0408)	1.0373 (1.2802)	0.4258 (.4258)	1.1061 (1.0773)
High	Medium	.9090 (.9929)	.8370 (.8370)	1.0381 (1.0263)	.9090 (.9853)	.8370 (.8370)	1.0374 (1.0263)
High	Low	1.1020 (1.1319)	.8442 (.6600)	1.0180 (1.0504)	1.0921 (.9895)	.6600 (.6600)	1.0576 (1.0662)
Medium	High	.8148 (.8132)	2.1999 (2.1999)	.9710 (.9710)	.7432 (.8093)	2.4124 (2.1999)	.9710 (.9710)
Medium	Medium	1.2051 (1.1529)	1.0206 (1.0206)	.9621 (.9704)	1.2051 (1.1529)	1.0206 (1.0206)	.9621 (.9704)
Medium	Low	.9363 (1.0875)	.9833 (.9833)	1.0203 (.9816)	.9300 (.9745)	.9833 (.9833)	1.0219 (1.0056)
Low	High	1.4230 (1.6958)	4.2740 (4.2740)	.8655 (.8428)	1.4230 (1.6869)	4.2740 (4.2740)	.8655 (.8428)
Low	Medium	1.1472 (1.4463)	1.1294 (1.1294)	.9305 (.8710)	1.1472 (1.4463)	1.1294 (1.1294)	.9304 (.8710)
Low	Low	.9339 (.9321)	1.1023 (1.1023)	.9858 (.9858)	.9272 (.9321)	1.1023 (1.1023)	.9882 (.9858)

Table 12.—Ratios of "local planning unit" output solution values occurring in both upper level Models 2A and 4 to those of the global model (medium global budget constraint and medium relative timber prices).

State	Ownership (NFS = National Forest System)	Upper level linear programming solution		Upper level integer programming solution	
		Timber	Forage	Timber	Forage
Idaho	NFS Other	1.1722 1.0010	1.2285 1.0000	1.1722 1.0010	1.2285 1.0000
Wyoming	NFS Other	1.2646 2.7601	1.1770 .9651	.7445 3.4869	.8199 .9756
Utah	NFS Other	1.0071 1.3025	.9996 .9855	1.0071 1.3025	.9996 .9855
Nevada	NFS Other	1.0000 1.0000	1.0012 1.0240	1.0000 1.0000	1.0012 1.0240
California	NFS Other	1.2032 1.1341	.9940 .9996	1.2032 1.1341	.9940 .9996
Oregon	NFS Other	1.0980 .6333	.9526 1.1496	1.0980 .6333	.9526 1.1496
Washington	NFS Other	1.0442[1] 1.0120	0.9507[1] 1.3317	1.0819 1.0120	.9342 1.3317

[1]*Local planning unit where partial selection occurred in upper level linear programming solution.*

Table 13.—Ratios of local planning unit cost allocations in both upper level Models 2A and 4 to those in the global model and lower level budget code selected[1] (medium global budget constraint and medium relative timber prices).

State	Ownership (NFS = National Forest System)	Upper level linear programming solution		Upper level integer programming solution	
		Cost allocation	Selected budget	Cost allocation	Selected budget
Idaho	NFS	1.4276	3	1.4276	3
	Other	1.0005	3	1.0005	3
Wyoming	NFS	1.4326	3	.7163	1
	Other	1.3745	3	1.6920	4
Utah	NFS	1.0069	1	1.0069	1
	Other	1.0628	3	1.0628	3
Nevada	NFS	1.0044	3	1.0044	3
	Other	1.0883	3	1.0883	3
California	NFS	1.0641	3	1.0641	3
	Other	1.0077	3	1.0077	3
Oregon	NFS	1.1561	3	1.1561	3
	Other	.6019	1	.6019	1
Washington	NFS	1.1198	.13(1) + .87(3)	1.1956	3
	Other	1.0716	3	1.0716	3

[1]*The lower level price code 3 was selected for all local planning units, in all solutions with the medium global budget constraint and medium relative timber prices.*

Table 14.—Ratios of a selected local planning unit (Idaho National Forest System) output and cost solution values in upper level linear (integer) programming Models 2A and 4 to those of the global model.

Global budget constraint	Relative timber price	Model 2A			Model 4		
		Timber	Forage	Cost (budget)	Timber	Forage	Cost (budget)
High	High	0.9576 (.9576)	0.2169 (.2169)	0.8750 (.8750)	0.8772 (1.0513)	2.0877 (2.1371)	0.8750 (1.3111)
High	Medium	.8955 (.8955)	.9771 (.9771)	.8057 (.8057)	.8955 (.8955)	.9771 (.9771)	.8057 (.8057)
High	Low	1.0369 (1.0369)	1.0268 (1.0268)	1.2210 (1.2210)	1.1912 (.9940)	.8803 (.8599)	1.2210 (.8149)
Medium	High	.8378 (.7376)	.4277 (.2876)	.7141 (.5299)	.6756 (.6756)	4.1356 (4.1356)	.5299 (.5299)
Medium	Medium	1.1722 (1.1722)	1.2285 (1.2285)	1.4276 (1.4276)	1.1722 (1.1722)	1.2285 (1.2285)	1.4276 (1.4276)
Medium	Low	.8744 (1.0483)	.9984 (1.0286)	.8327 (1.2476)	1.0049 (1.0049)	.8614 (.8614)	.8327 (.8327)
Low	High	.9074 (.9074)	.7279 (.7279)	.8443 (.8443)	.8311 (.8311)	10.4659 (10.4659)	.8443 (.8443)
Low	Medium	1.0722 (1.4920)	1.0570 (1.6987)	1.1291 (2.2582)	1.0722 (1.4920)	1.0570 (1.6987)	1.1291 (2.2582)
Low	Low	.9797 (1.0782)	.9897 (1.0295)	1.0209 (1.1432)	1.2391 (1.2391)	.8882 (.8882)	1.1432 (1.1432)

lesser degree, this also shows up in table 12. This sort of instability is somewhat discouraging and again points to the need for careful interpretation of the multilevel model solutions at the local level. Because the global solution generally would not be known, this evaluation admittedly would be subjective and unreliable —especially if the standard of comparison tended to be local optima, which may have no direct relationship with the global optima.

Tables 12 and 13 indicate the potential instabilities that could occur in even a relatively optimistic situation. To indicate a more complete picture of the potential local planning unit instabilities, table 14 presents the solution value ratios for outputs and costs with all nine budget and price vector combinations, for a selected (representative) local planning unit. The solution values for the given local planning unit from the multilevel model differ from the global solution values by as much as a factor of 10. The local solution values in the multilevel model also appear to be particularly unstable when the extreme relative timber prices are taken in combination with one of the more extreme global budget constraints.

Conclusions

The (Bartlett-Wong) multilevel modeling approach tested here does not appear to closely emulate a global optimization in terms of local level output solution values or budget allocations across local planning units. Regional output solution values are closely approximated only in the more optimistic cases. Yet, with moderate relative prices, the multilevel model comes very close to the global model in terms of total output solution values and, especially, in terms of overall optimality. In the tests discussed, the objective functions were a variety of weighted summations of outputs; therefore, stating that optimality is closely approximated implies that the multilevel model is able to locate "points" that are close to the true production possibilities frontier implied in the global model.

The only obvious way to interpret these results is that there are, in the global model tested, many ways to alter local level solutions that do not significantly affect global optimality (as measured by the given objective function). This implies that the multilevel model may be tenable in analyzing national production possibilities, but may lead to local level resource allocations that are not truly efficient relative to the local resource allocations that would result from a single-level global optimization. It is apparent that the multilevel optimization approach tested here could lead to undiscovered resource misallocations at the lower levels of the given planning system. Again, the multilevel approach is only suggested for cases where a single-level global optimization is not pragmatically possible.

Given these conclusions, and the fact that optimization problems that cannot be handled with a single, global model are likely to persist, it would appear that renewable resource agencies might consider developing the capability to pursue the Kornai and Liptak (1965) approach. The Bartlett-Wong approach evaluated here shows impressive capabilities at the highest level of the model; but because the allocations to lower levels seem to be suspect, it is not a universal solution. To employ a model along the lines suggested by Kornai and Liptak, the agency involved would need to have all lower level models operational during the same period of time, and communication channels would have to be opened to support the iterations between higher and lower levels. Until then, the Bartlett-Wong approach seems to have the potential to reliably address at least part of the problem, and encourages the general sorts of lower level models and communication lines that would be necessary for the Kornai and Liptak approach.

In applying the Bartlett-Wong approach, the findings in this report would suggest first, that the integer programming approach appears to perform very nearly as well as the linear programming approach (in the upper level model) in terms of approximating global optima. Because it treats lower level alternatives discretely, it is more reliable. An approach has been presented that allows solution of relatively large integer programs; but if a great many alternatives are included, an integer solution approach may not be possible. In this case, the linear programming approach to the upper level model appears to be a reasonable alternative.

Second, it is important to the performance of the Bartlett-Wong approach that the local planning units develop alternatives with prices and other variables (that are not used in global constraints) as close as possible to those that will eventually be used in the upper level model. This means that the direction of local planning analyses would not be toward defining the absolute limits of production or toward generating the widest possible range of alternatives. Instead, it would mean that the more information the national planning authority can provide to the local planners ahead of time, the more the local alternatives could focus on the relevant range of alternatives, and the more effective the Bartlett-Wong approach would be.

Third, if one purpose of local planning efforts is to support an analysis such as the Bartlett-Wong model, then the results reported here suggest that systematic variation in the implied budget requirements across local alternatives is more important than determining a variety of output sets on a given production frontier. This conclusion almost certainly results from the fact that the test case was formulated to address the problem of allocating a global budget to local planning units. If a different input or output constraint (or group thereof) is common to the local planning units, then parallel conclusions are likely to be drawn. If local planning units do not generate alternatives in a systematic manner, then there is no guarantee that the needed variation in the key variables will exist in the resultant alternatives. With this variation in key variables, the Bartlett-Wong approach appears to perform well (at the national level), even with a rather limited number of alternatives for each local planning unit.

Literature Cited

Ashton, P. G., J. B. Pickens, C. Ohlander, and B. Benninghoff. 1980. Many resources, many uses: A system analysis approach to renewable resource development. Water Resources Bulletin 16:738–744.

Bartlett, E. T. 1974. A desision-aiding model for planning optimal resource allocation of water basins. Ph.D. dissertation, 132 p. University of Arizona, Tucson.

Dantzig, G. B., and P. Wolfe. 1961. The decomposition algorithm for linear programs. Econometrica 29: 767–778.

Dantzig, G. B. 1963. Linear programming and extensions. 627 p. Princeton University Press, Princeton, N. J.

D'Aquino, S. A. 1974. Optimum allocation of resources: A programming approach. Journal of Range Management 27:228–233.

Hof, J. G., R. K. Marose, and D. A. King. 1985. Potential pitfalls in renewable resource decision making that utilizes convex combinations of discrete alternatives. Western Journal of Agricultural Economics 10:391–400.

Kornai, J., and T. Liptak. 1965. Two-level planning. Econometrica 33:141–169.

Kornai, J. 1975. Mathematical planning of structural decisions. 644 p. American Elsevier Publishing Company, New York, N. Y.

Kuchler, A. W. 1965. Potential natural vegetation of the coterminous United States (manual and map). American Geographic Society Special Publication 36(Rev.), 116 p., New York..

Silberberg, E. 1978. The structure of economics: A mathematical analysis. 543 p. McGraw-Hill, New York, N. Y.

USDA Forest Service. 1980. A recommended renewable resources program—1980 update. U.S. Department of Agriculture, Forest Service Report FS–316, 540 p. Washington, D.C.

Wong, C. P. 1980. A multilevel approach to the forest service planning process. M.S. thesis. Colorado State University, Fort Collins.

Appendix 1

Detailed Description of Global Model (NIMRUM)

The land classification system used in NIMRUM was based on Kuchler's (1965) Potential Natural Vegetation scheme with the addition of one vegetation type that Kuchler did not include. This type, "mountain meadows," was included because of its large impact on western range production. To avoid confusion, the land classifications were renamed "potential natural communities" (PNC). Within each PNC, the land base was separated by ownership, productivity class, and condition class designations (table A1). Each of these designations had four possible classifications, giving a total of 64 possible resource units (RU) for each PNC. The RU was the basic land management unit for the NIMRUM model. These RU's are analagous to the TYPE I and II lands in table 1. Outputs and costs per acre were assumed to be the same for an RU independent of where the specific land parcel was located.

The management options for each RU were defined as a combination of practices seeking a specific level of management (ML) for each of three output groups. The output groups were timber, range, and wildlife. Timber has six possible ML's (1 to 6), range has five possible ML's (A to E), and wildlife has three possible ML's (1 to 3). The definition of each of these levels is briefly presented in table A2. A combination of three ML's was called a management triplet. A subset of the 73 possible management triplets was selected by the interdisciplinary teams for each RU. E level range management excludes, by definition, any active management for the other outputs. In general, the ML triplets were selected for each PNC and then for each RU. Thus, RU's within a PNC had the same options or a subset of the options chosen for that PNC. Most PNC's had about 10 management triplets specified.

After the management level triplets had been selected, specific practices were selected for each RU to attain each desired management triplet. The management triplets were simply used to organize management op-

Table A1.—Land classification system within a PNC.

Code		
Ownership Code	**Ownership**	
1	National Forest System	
2	Natural Resource Lands (BLM)	
3	Other Federal	
4	Non-Federal	
Productivity Class Code	**Forested PNC's**	**Non-Forested PNC's**
1	120 + ft.^{3}/ac./yr.	High
2	85-110 ft.^{3}/ac./yr.	Moderately High
3	50-84 ft.^{3}/ac./yr.	Moderately Low
4	0-49 ft.^{3}/ac./yr.	Low
Condition Class Code	**Forested**	**Non-Forested**
1	Non-stocked	Good
2	Seedlings and Saplings	Fair
3	Poles	Poor
4	Sawtimber	Very Poor

Table A2.—Management level designations.

Range Management Level

A - Environmental management without livestock
B - Environmental management with livestock
C - Extensive management of environment and livestock
D - Intensive management of environment and livestock
E - Maximum management of environment and livestock

Timber Management Level

1 - No commercial use
2 - Minimal management (high-grading)
3 - Harvest and assure regeneration
4 - Harvest, assure regeneration, and commercial thin
5 - Harvest, assure proper stocking, and competition removal
6 - Harvest, assure proper stocking and competition removal, and fertilizing, and/or tree improvement

Wildlife Management Level

1 - No management
2 - Vegetation manipulation
3 - Vegetation manipulation and placement of structures

tions; these designations are somewhat artificial because of the jointness of the renewable resource production system. That is, practices designated for one output type actually affect the production of other outputs. The practices allowed are presented in table A3 by output classes.

After each management triplet was defined by specific practices, the interdisciplinary team predicted the quantity of outputs that would be expected. These formed the columns of the linear programming A-matrix (analagous to table 1). There were 13 outputs in NIMRUM; but only two outputs were included in this study—cubic feet of wood harvested and animal unit months of domestic forage grazing.

The NIMRUM model does not address scheduling problems directly. Information was collected on the amount of time that an RU with a given management triplet would be in specified condition classes. The practices and outputs data originally generated were the average levels over the condition classes. A program was written to convert the production and cost information from average per year for the condition classes into averages based on a 50-year planning horizon. A particular 50-year schedule is assumed for each management triplet in each RU. Thus, NIMRUM is an average-year static model. This planning horizon was chosen because the legislative mandate for RPA specifies a 50-year planning period.

Table A3.—Management practices by target output group.

	Units/100 M acres
Range Management Practice	
Fertilization	M acres
Irrigation	M acres
Water control	M acres
Mechanical vegetation control (low cost)	M acres
Mechanical vegetation control (high cost)	M acres
Vegetation manipulation - chemical	M acres
Vegetation manipulation - biological	M acres
Vegetation manipulation - fire	M acres
Debris disposal	M acres
Mechanical soil treatment	M acres
Seeding	M acres
Rodent control	M acres
Insect and disease control	M acres
Small water development	Sites
Large water development	Sites
Fence	Miles
Timber thinning	M acres
Timber Management Practice	
Planting	M acres
Direct seeding	M acres
Site prep. for natural regeneration	M acres
Site prep. for planting and seeding	M acres
Animal control for reforestation	M acres
Animal control for timber stand improvement	M acres
Precommercial thin	M acres
Release and weeding	M acres
Fertilization of established stands	M acres
Seed production areas	M acres
Selection and care of superior trees	M acres
Prescribed burning to control understory	M acres
Access roads for timber production	Miles
Cutting method-shelterwood and seed tree	M acres
Cutting method-salvage	M acres
Cutting method-clearcutting	M acres
Cutting method-commercial thinning	M acres
Cutting method-selection	M acres
Cutting method-selective	M acres
Wildlife Management Practice	
Water developments - upland	Sites
Seeding and planting	M acres
Liming and fertilizing	M acres
Fencing	Miles
Prescribed burning - uplands	M acres
Clearing	M acres
Brush and shrub management-mechanical	M acres
Brush and shrub management-chemical	M acres
Brush and shrub management-biological	M acres
Pruning	M acres
Thinning - release	M acres
Mechanical soil treatment	M acres
Dens and nest structures	Structures
Perch and nest structures	Structures
Brushpiles and covers	Brushpiles
Streambank stabilizers	Sites

Appendix 2

Mathematical Description of the Test Case Models

The only pervasive difference among models is the portion and level of aggregation of the land base used. The objective function for all three models is the same, to maximize weighted outputs. The production constraints are analogous and are only used to transfer outputs to the objective function. The inputs, budget and land, are analogous in all three models.

The superscript asterisk indicates that upper level model land unit definitions are being used. Absence of the superscripted asterisk indicates that lower level model land definitions are used. For example, the subscript i identifies specific resource units in one of the local planning unit models. In contrast, i* represents the land base for one of the local planning units.

The algebraic representations along with definitions for subscripts and variables is:

Upper Level Model Algebraic Formulation

Maximize: $$\sum_{k=4}^{6} \sum_{p=1}^{2} W_{kp} \; T_{kp} \quad \text{(Objective function)}$$

Subject to: $$\sum_{i^*=1}^{I^*_k} \sum_{j^*=1}^{J^*} P_{i^*j^*p} \; X_{i^*j^*} - T_{kp} = 0 \;\forall k,p$$
(Production accounting rows)

$$\sum_{k=4}^{6} \sum_{i^*=1}^{I^*_k} \sum_{j^*=1}^{J^*} C_{i^*j^*} \; X_{i^*j^*} \leq B$$
(Budget constraint)

$$\sum_{j^*=1}^{J^*} X_{i^*j^*} = 1 \;\forall i^* \quad \text{(Unity constraints)}$$

Lower Level Model Algebraic Formulation

Maximize: $$\sum_{p=1}^{2} W_{kp} \; T_{kp} \quad \text{(Objective function)}$$

Subject to: $$\sum_{i=1}^{I_{i^*}} \sum_{j=1}^{J_{ii^*}} P_{ijp} \; X_{ij} - T_{kp} = 0 \;\forall p$$
(Production accounting rows)

$$\sum_{i=1}^{I_{i^*}} \sum_{j=1}^{J_{ii^*}} C_{ij} \, X_{ij} \leq B_{i^*} \quad \text{(Budget constraint)}$$

$$\sum_{j=1}^{J_{ii^*}} X_{ij} = L_i \;\forall i \quad \text{(Land area constraints)}$$

Global Model Algebraic Formulation

Maximize: $$\sum_{k=4}^{6} \sum_{p=1}^{2} W_{kp} \; T_{kp} \quad \text{(Objective function)}$$

Subject to: $$\sum_{i^*=1}^{I^*_k} \sum_{i=1}^{I_{i^*}} \sum_{j^*=1}^{J_{ii^*}} P_{ijp} \, X_{iji^*k} - T_{kp} = 0 \;\forall k, p$$
(Production accounting rows)

$$\sum_{i^*=1}^{I^*_k} \sum_{i=1}^{I_{i^*}} \sum_{j=1}^{J_{ii^*}} C_{ij} \, X_{iji^*k} \leq B$$
(Budget constraint)

$$\sum_{j=1}^{J_{ii^*}} X_{iji^*k} = L_{ii^*k} \;\forall i,i^*$$
(Land area constraints)

Definition of Variables

Subscripts

- i — Represents the resource units (unique land types) for the lower level models.
- i* — Represents the relevant lower level models for the k^{th} region.
- j — Represents the options for management of each resource unit.
- j* — Represents the solutions within each lower level model.
- k — Represents the Forest Service region, k = 4, 5, or 6.
- p — Represents the products considered by the model—wood harvest and domestic AUM production.

Variables

- B = The amount of budget available.
- B_{i^*} = The budget allocation for the i*th lower level model.
- C_{ij} = The cost of managing 1 acre of resource unit i under management level j.
- $C_{i^*j^*}$ = The cost for managing all of the lower level model i* under lower level solution j*.
- I_{i^*} = The number of resource units in lower level model i*.
- I^*_k = The number of lower level models in the k^{th} region.
- J_{ii^*} = The number of management levels available for land unit i in lower level model i*.
- J^* = The number of lower level solutions for each lower level model.
- L_i = The total number of acres of resource unit i.
- L_{ii^*k} = The number of acres available in resource unit i of lower level model i* and region k.
- P_{ijp} = The output of product p from resource unit i managed in management level j.
- $P_{i^*j^*p}$ = The output of product p from local planning unit i* under management j*.
- T_{kp} = A variable to transfer the output from region k for product p from the production accounting rows to the objective function.
- W_{kp} = The weight associated with a unit of product p produced in region k. These weights can be viewed as relative prices.
- X_{ij} = The number of acres of resource unit i in managment level j chosen by the model.
- $X_{i^*j^*}$ = The proportion of the land base for lower level planning unit i* that is allocated to lower level solution j*.
- X_{iji^*k} = The number of acres in the global model that were selected for management level j from resource unit i in lower level model i* of region k.

Hof, John G., and James B. Pickens. 1986. A multilevel optimization system for large-scale renewable resource planning. USDA Forest Service General Technical Report RM–130, 23 p. Rocky Mountain Forest and Range Experiment Station, Fort Collins, Colo.

This report analyzes and evaluates one approach to utilizing local planning analyses of public renewable resource management agencies as an input in developing large-scale (national) resource management plans. A type of multilevel approach (referred to as the Bartlett-Wong approach) is discussed, and is evaluated in a test case. This prototype performed well at the highest level of analysis but was less reliable in terms of its implications for the lower level planning units.

Keywords: Land management planning, forest economics, linear programming, integer programming.

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U.S. Department of Agriculture
Forest Service

Rocky Mountain Forest and Range Experiment Station

Rocky Mountains

Southwest

Great Plains

The Rocky Mountain Station is one of eight regional experiment stations, plus the Forest Products Laboratory and the Washington Office Staff, that make up the Forest Service research organization.

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Research programs at the Rocky Mountain Station are coordinated with area universities and with other institutions. Many studies are conducted on a cooperative basis to accelerate solutions to problems involving range, water, wildlife and fish habitat, human and community development, timber, recreation, protection, and multiresource evaluation.

RESEARCH LOCATIONS

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Albuquerque, New Mexico
Flagstaff, Arizona
Fort Collins, Colorado*
Laramie, Wyoming
Lincoln, Nebraska
Rapid City, South Dakota
Tempe, Arizona

*Station Headquarters: 240 W. Prospect St., Fort Collins, CO 80526